Math 42 Midterm 2 Review

You will need a calculator to solve the problems marked ★.
You should **NOT** use a calculator for any other problems.

[0] Print out the inverse trigonometric values "flashcard" from my website, and cut into squares.
Randomly select an inverse trigonometric function "flashcard" and a trigonometric value "flashcard".
Identify the value of the inverse trigonometric function as applied to the trigonometric value.
Some combinations will generate values that can only be found using a calculator,
while other combinations will not generate any value. You should be able to identify those.
Repeat as many times as necessary to master the inverse trigonometric function values.

[1] Identify the middle value, amplitude, period and phase shift.
Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete cycles, starting at the phase shift.
Sketch a detailed graph of 2 complete cycles using the information you found.
Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.
Also, write the equation and sketch the graph of the corresponding reciprocal function.

[a] $y = 2\sin(\frac{1}{2}x - \frac{5\pi}{4}) - 3$
[b] $y = -3\cos(\frac{3\pi}{4}x - \frac{\pi}{2}) - 4$
[c] $y = -5\sin(\frac{5\pi}{3}x + \frac{2\pi}{9}) + 1$
[d] $y = 4\cos(3x + \frac{5\pi}{3}) + 2$

[2] Sketch the graphs. **You only need to get the general position and shape correct. Do NOT plot points.**
Find the domain, range, and equations of all asymptotes.

[a] $y = \sec x$
[b] $y = \csc x$
[c] $y = \cot x$
[d] $y = \tan x$
[e] $y = \sin^{-1} x$
[f] $y = \cos^{-1} x$
[g] $y = \tan^{-1} x$

[3] Fill in the blanks. Use the graphs from [2].

[a] As $x \to -\frac{\pi}{2}^+$, $\sec x \to$ _____ and $\tan x \to$ _____
[b] As $x \to 1^-$, $\cos^{-1} x \to$ _____
[c] As $x \to -\pi^+$, $\csc x \to$ _____ and $\cot x \to$ _____
[d] As $x \to \infty$, $\tan^{-1} x \to$ _____
[e] As $x \to \frac{3\pi}{2}^-$, $\sec x \to$ _____ and $\tan x \to$ _____
[f] As $x \to -1^+$, $\sin^{-1} x \to$ _____
[g] As $x \to 0^-$, $\csc x \to$ _____ and $\cot x \to$ _____
[h] As $x \to -\frac{3\pi}{2}^+$, $\sec x \to$ _____ and $\tan x \to$ _____
[i] As $x \to -1^+$, $\cos^{-1} x \to$ _____
[j] As $x \to 0^+$, $\csc x \to$ _____ and $\cot x \to$ _____
[k] As $x \to -\infty$, $\tan^{-1} x \to$ _____
[l] As $x \to \frac{\pi}{2}^-$, $\sec x \to$ _____ and $\tan x \to$ _____
[m] As $x \to 1^-$, $\sin^{-1} x \to$ _____
[h] As $x \to \pi^-$, $\csc x \to$ _____ and $\cot x \to$ _____

[4] Simplify the following expressions. **Some expressions have no value.**

[a] $\sin(\sin^{-1}\frac{2}{3})$ [b] $\tan^{-1}(\tan\frac{\pi}{6})$ [c] $\cos^{-1}(\cos(-\frac{\pi}{3}))$ [d] $\sin^{-1}(\sin(-\frac{\pi}{4}))$

[e] $\cos^{-1}(\cos\frac{5\pi}{6})$ [f] $\sin^{-1}(\sin\frac{2\pi}{3})$ [g] $\tan(\tan^{-1}(-\frac{5}{4}))$ [h] $\cos(\cos^{-1}\frac{4}{3})$

[i] $\tan(\tan^{-1}\frac{4}{5})$ [j] $\cos(\cos^{-1}(-\frac{3}{4}))$ [k] $\sin(\sin^{-1}(-\frac{3}{2}))$ [l] $\tan^{-1}(\tan(-\frac{3\pi}{4}))$

[5] Simplify the following expressions. **Some expressions have no value.**

[a] $\sin(\tan^{-1}\frac{2}{3})$ [b] $\sec(\sin^{-1}(-\frac{3}{4}))$ [c] $\csc(\cos^{-1}\frac{3}{2})$ [d] $\cos(\sin^{-1}\frac{4}{5})$

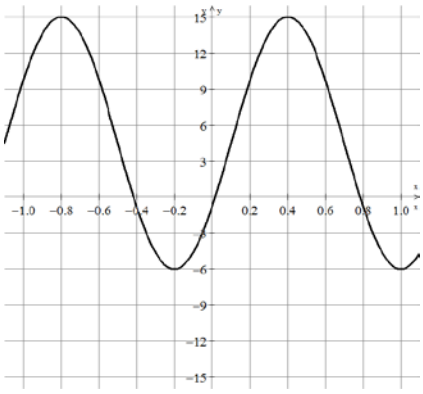
[e] $\tan(\cos^{-1}\frac{1}{3})$ [f] $\cot(\sin^{-1}(-\frac{4}{3}))$ [g] $\csc(\tan^{-1}(-2))$ [c] $\cot(\cos^{-1}(-\frac{2}{5}))$

[6] Simplify the following expressions.

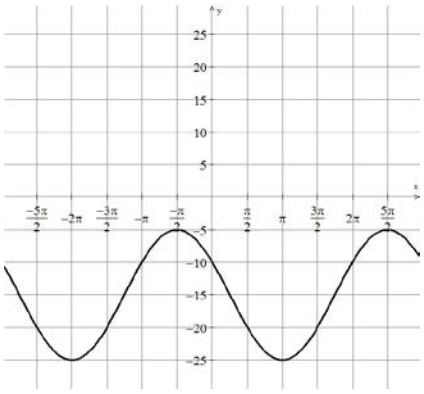
[a] $\tan(\sin^{-1}(x+1))$ [b] $\cos(\tan^{-1}\frac{t}{2})$ [c] $\sin(\cos^{-1}\frac{\sqrt{1-y}}{2})$

[7] Find an equation for each graph.

[a]



[b]



[8] AJ has been reading about biorhythms, and decided to make a chart of his overall mood starting on Jan 1. On a scale of 1 to 10, AJ's mood reached a high of 9.5 on Jan 17, and dropped continually to a low of 2 on Feb 2, before starting to rise again. Assume AJ's mood corresponds to a sinusoidal function.

[a] Find an equation for his mood on the t^{th} day since Jan 1.
[b]★ What will AJ's mood be on Mar 19 ?

[9]★ A 265 meter cable connects the roofs of two buildings.
From the roof of the first building, the angle of depression to the roof of the second building is 41°.
From the base of the first building, the angle of elevation to the roof of the second building is 67°.

[a] Find the height of each building, and the distance between the bases of the buildings.
[b] Find the angle of elevation of the roof of the first building from the base of the second building.

[10]★ A 17 foot ladder is leaning against the wall of a building. The base of the ladder is 8 feet from the base of the building.

[a] Find the angle between the ladder and the building.
[b] Find the angle between the ladder and the ground.

[11] If a ferris wheel is turning at a constant rate, then the height of a particular seat relative to the center of the wheel is a form of simple harmonic motion.

Consider a ferris wheel of radius 65 feet, which takes 8 seconds for a seat to go from the bottom of the wheel to the top.
Assume the height of a seat is considered positive if it is above the center of the ferris wheel, and negative if it is below.

[a] What is the frequency of the wheel ?
[b] Find the equation of motion of a seat which starts at the bottom of the wheel.
[c] Find the equation of motion of a seat which is directly to the right of the center of the wheel, if the wheel is turning clockwise.

[12]★ A car is travelling at 57 miles per hour directly southward.

At 2pm, the car is 81 miles from the center of a town, on a bearing of 281° from the center.
At 2:30pm, the car turns and begins travelling directly eastward at the same speed.

[a] At 2pm, how far is the car north/south and east/west of the center of town ?
[b] At 2:30pm, how far is the car north/south and east/west of the center of town ?
[c] At 2:30pm, how far is the car from the center of town, and on what bearing from the center ?
[c] At what time will the car be directly southeast of the center of town ?